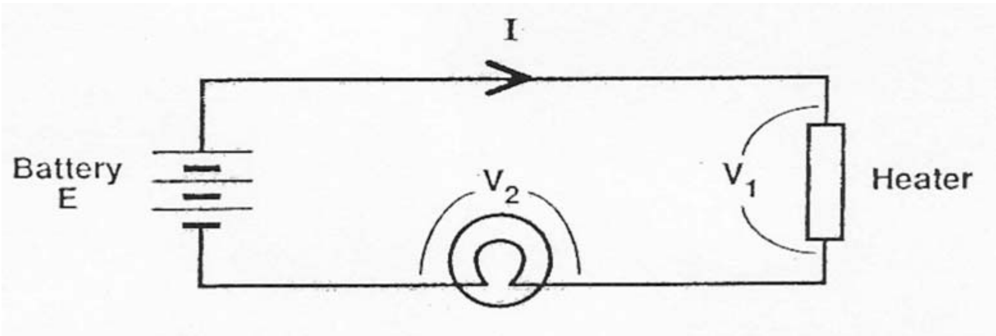
Electrical Circuit Calculations

Series Circuits

Many circuits have more than one conversion device in them
(i.e. toaster. heater. lamps etc.) and some have more than one source of electrical energy.

If the circuit components are connected end to end to form a single loop it is a series-circuit



Remember that current is the rate at which electrons move through the circuit. So as in several hoses connected in series to form one long line, water can only come out at the far end at the same rate that it enters at (neglecting friction).

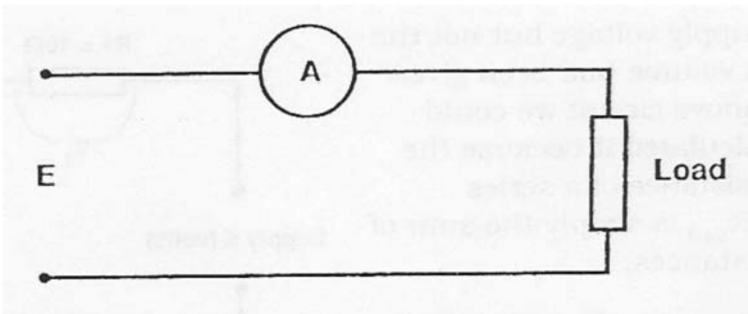
Each component has a volt drop across it (and is the force required to convert the electrical energy to some other form). All of the volt drops in a circuit add up to the e.m.f. (Voltage) of the supply.

In the above circuit $E = V_1 + V_2$

(This assumes that the connecting wires have very little resistance, which is not an unreasonable assumption to make.)

Ammeters

Ammeters are instruments that measure current and so have to be in the current path, that is, in series with the device whose current is being measured.



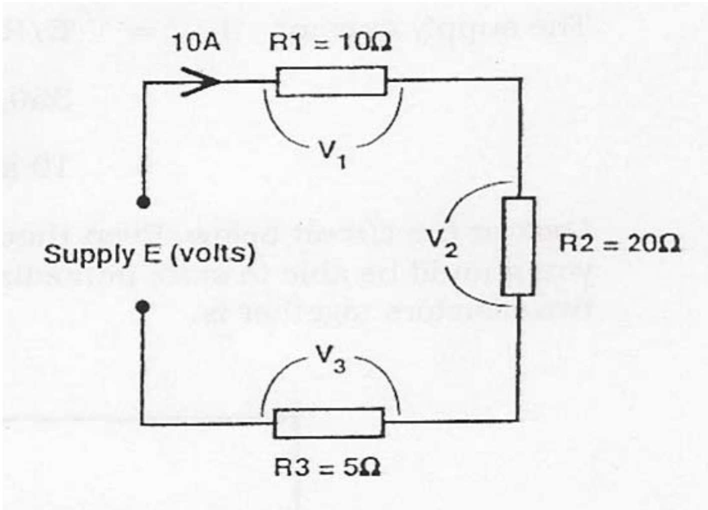
The volt drop across each component can be calculated using Ohms law if the circuit current and the value of each component's resistance is known.

(Remember that the e.m.f of the supply is across the whole circuit and not just across anyone of the components.)

Worked Example:

- If the circuit shown draws 10 amps from the supply, calculate the volt drop across each resistor and the volume of the supply e.m.f.

Note that as it is a series circuit the current is the same in all resistors.



$V_1 = IR_1 = 10 \times 10 = 100V$

$V_2 = IR_2 = 10 \times 20 = 200V$

$V_3 = IR_3 = 10 \times 5 = 50V$

As the volt drops all add up to the supply voltage E

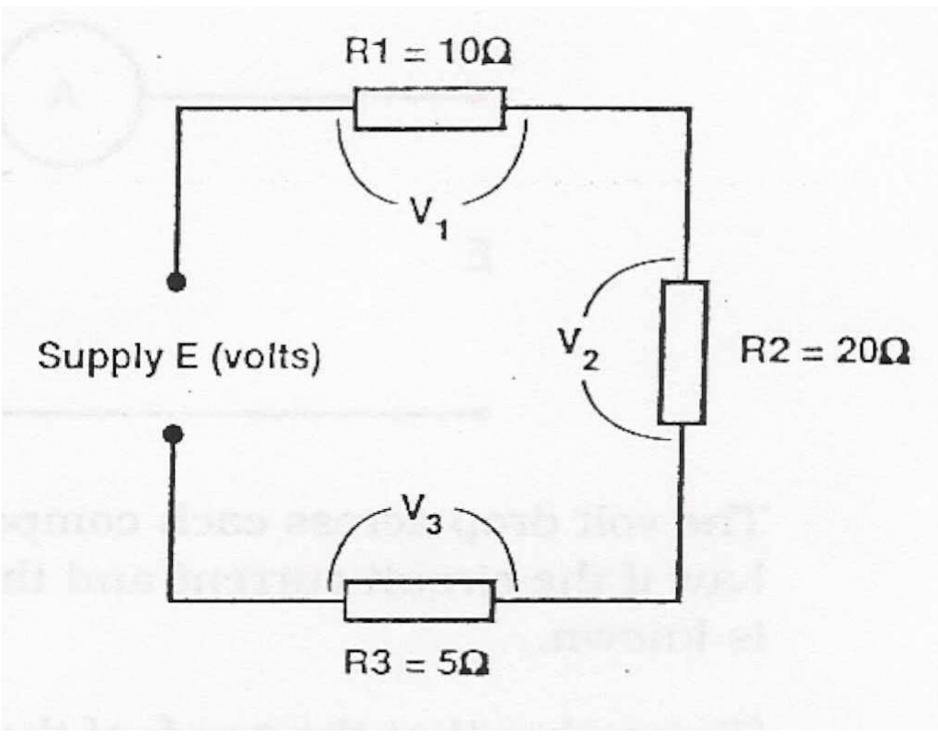
$E = V_1 + V_2 + V_3$

$= 100+200+50$

$= 350v$

If the supply voltage but not the current volume had been given in the above circuit we could have calculated it because the total resistance of a series circuit R_{total} is simply the sum of the resistances

This is the total resistance across the supply so this circuit can be replaced by a single resistor of 35 ohms.

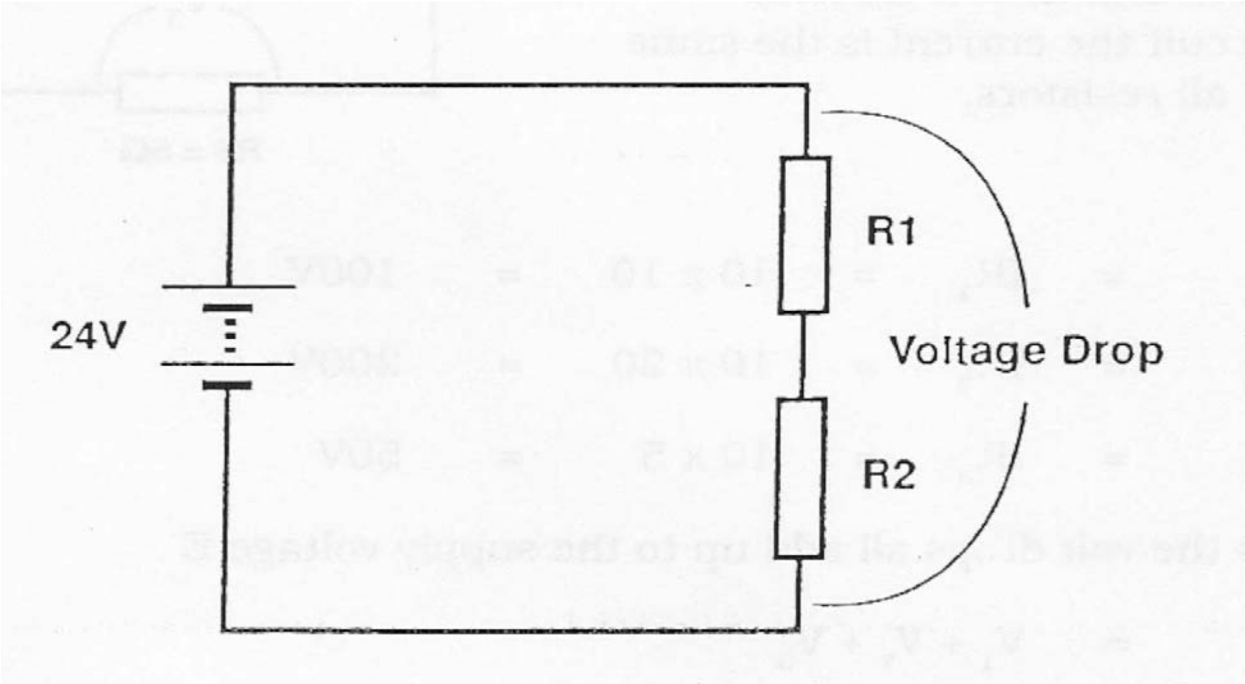


The supply current $I = E/R_t$

$= 350/35$

$= 10$ amps

Look at the circuit below. Even though we have connected two resistors you should be able to state immediately what the volt drop across the two resistors together is.



The voltage drop is, of course, 24 volts.

If the supply e.m.f E and the separate resistances are given, calculate the circuit current and the volt drops across each resistor.

1 Add the resistances to give the total resistance

2 Calculate the circuit current by dividing the supply E by R_T

3 Calculate the volt drop across each resistor by using the formula:

$V_1 = I \times R_1$

$V_2 = I \times R_2$

$V_3 = I \times R_3$ etc.

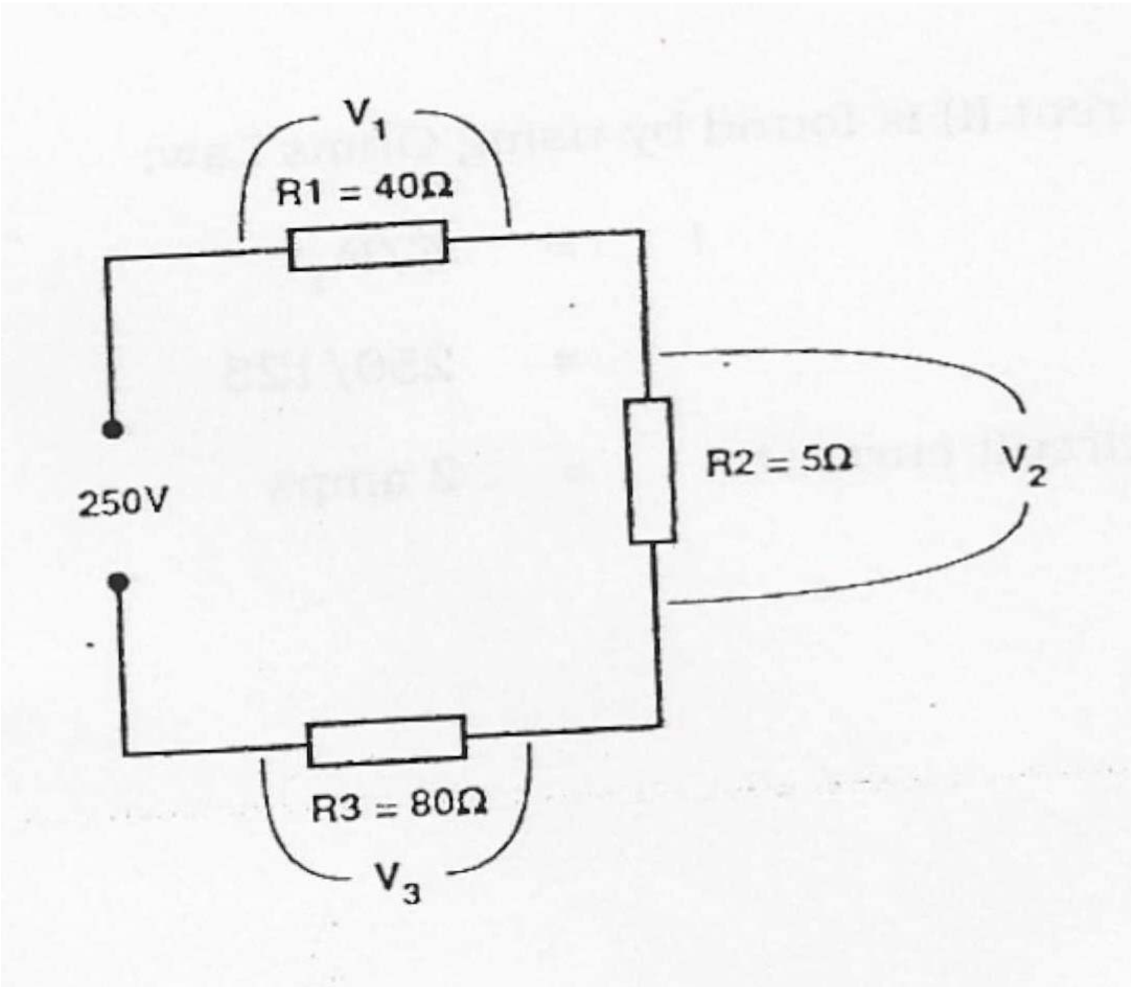
A Worked Example

- Given the circuit below calculate

  I The total resistance

  2 The current.

  3 The volt drop across each resistor-



1. Total resistance: $R_{total} = R_1 + R_2 + R_3$

   $= 40 + 5 + 80$

   $= 125$ ohms

2. Current (I) is found by using Ohms Law: $I = E/R_T$

   $= 250/125$

Therefore, circuit current I = 2amps

(3) Volt drop across each resistor:

Volt drop across R1: $V_1 = I \times R_1$

$= 2 \times 40$

$= 80$ volts

Volt drop across R2: $V_2 = I \times R_2$

$= 2 \times 5$

$= 10$ volts

Volt drop across R3: $V_3 = I \times R_3$

$= 2 \times 80$

$= 160$ volts

As a check, the volt drops across the resistors should add up to the supply voltage.

i.e $= VD_1 + VD_2 + VD_3$

$= 80+10 + 160$

$= 250$ volts.

Parallel Circuit

When each of a number of circuit components is connected across the same supply or between the same two points in a circuit, they are said to be connected in parallel.

Figure 1

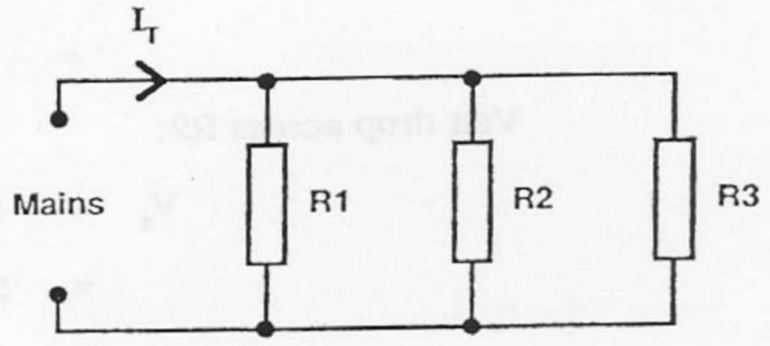
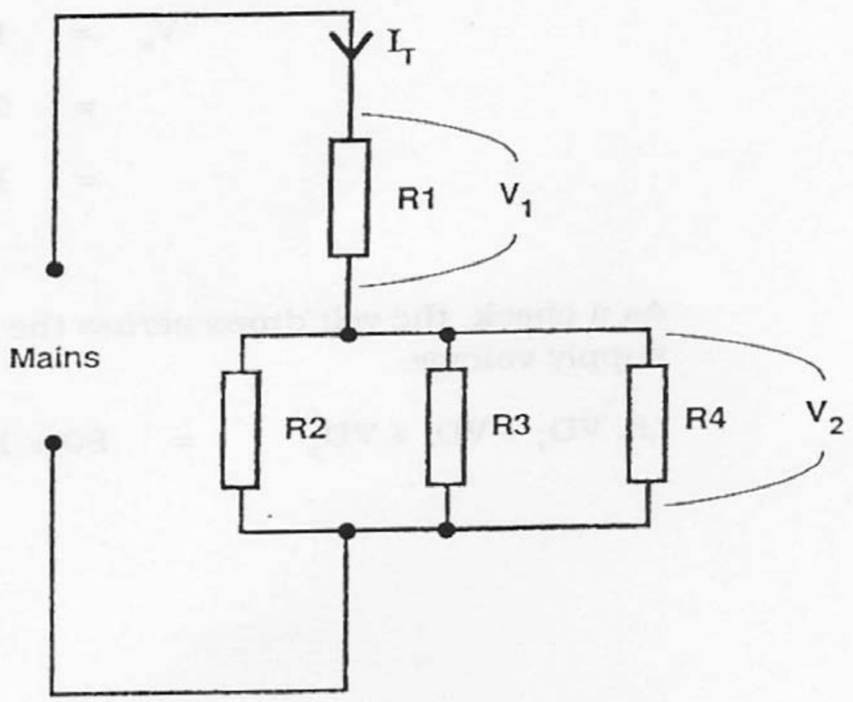


Figure 2

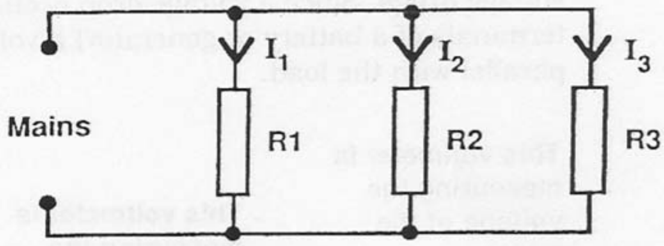


Whenever resistors are in parallel there is the same volt drop across each one. In figure one the volt drop across R1, R2 and R3 will be the same and will be the supply voltage. In figure two the volt drop across E2, R3 and R4 will be the same and it will be V_2

Unlike the series connection the current in resistors in parallel is not the same (unless they are the same value of resistance).

In figure 1 the supply current I_T divides into I_1, I_2 and I_3 as shown

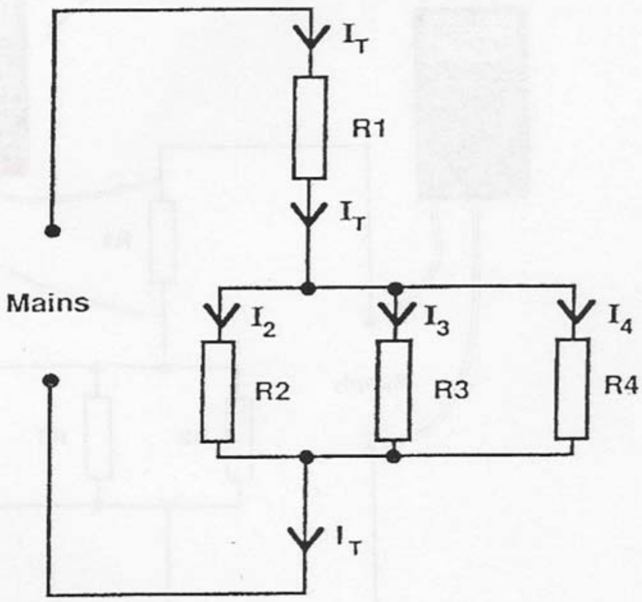
Figure 1



$\therefore I_T = I_1 + I_2 + I_3$

below, and in figure 2:

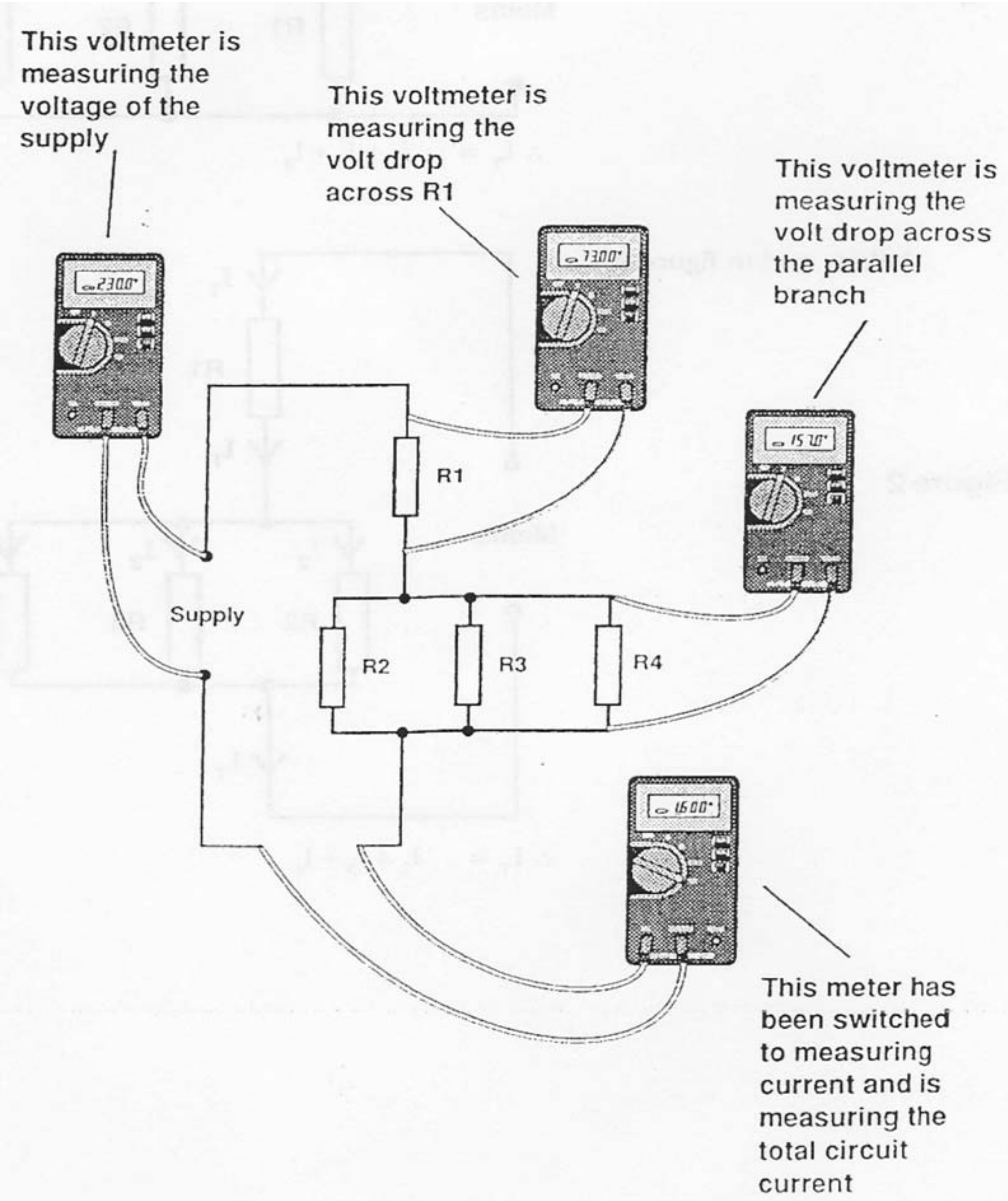
Figure 2



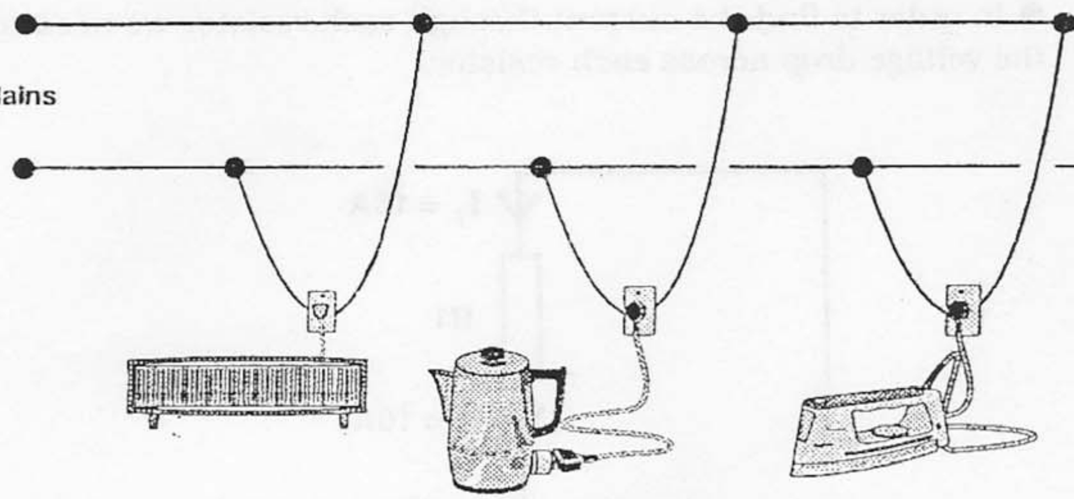
$\therefore I_T = I_2 + I_3 + I_4$

Voltmeters

Voltmeters are instruments used to measure voltages either e.m.f.s or voltage drops. Since a voltage drop occurs across a resistor (or the terminals of a battery or generator) a voltmeter is always connected in parallel with the load

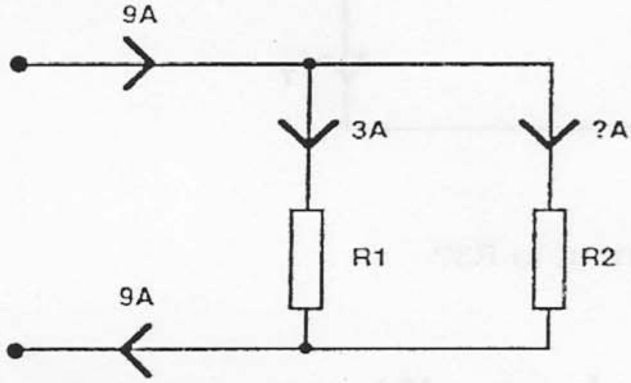


Most domestic appliances are connected in parallel.



Worked Example

- What is the current in R2 in this circuit below?



The total current through a junction equals the sum of the currents through the circuits leaving the junction.

$I_T = 9A$, $I_1 = 3A$, $I_T = I_1 + I_2$

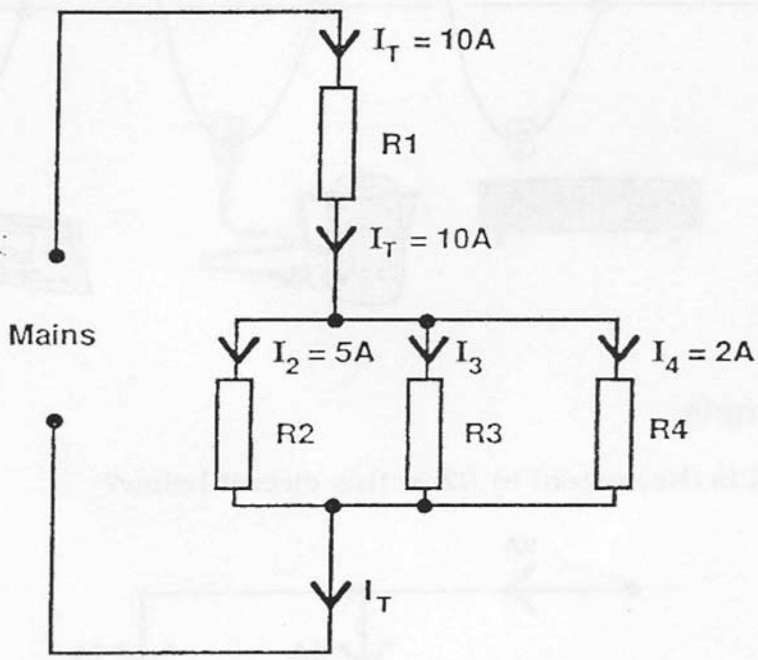
Therefore: $9A = 3A + I_2$

$I_2 = 9 - 3$

$= 6$ amps

Another Worked Example

- In order to find the current through each resistor we need the voltage drop across each resistor.



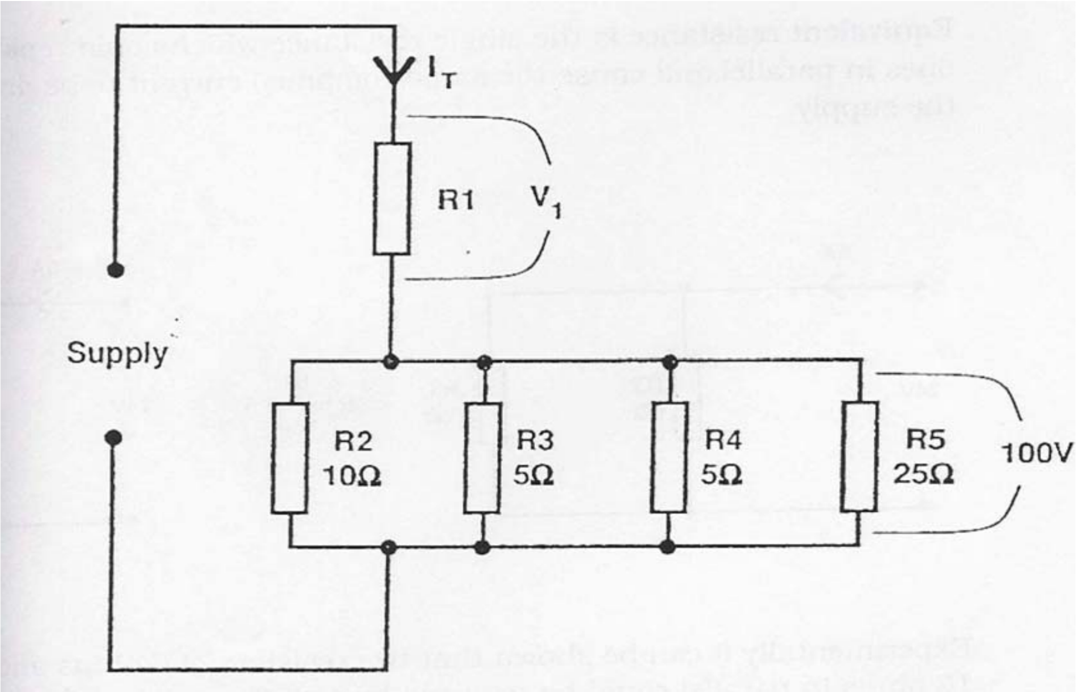
What is the current in R3?

Answer:

$$I_T = 10A$$
$$I_T = I_2 + I_3 + I_4$$
Therefore $10 = 5 + I_3 + 2$
$$10 - 7 = I_3$$
$$I_3 = 3A$$

Another Worked Example

- Study this circuit:



Calculate the volume of the current in R1.

$$I_2 = \frac{V_2}{R_2} = \frac{100}{10} = 10A$$

$$I_3 = \frac{V_2}{R_3} = \frac{100}{5} = 20A$$

$$I_4 = \frac{V_2}{R_4} = \frac{100}{5} = 20A$$

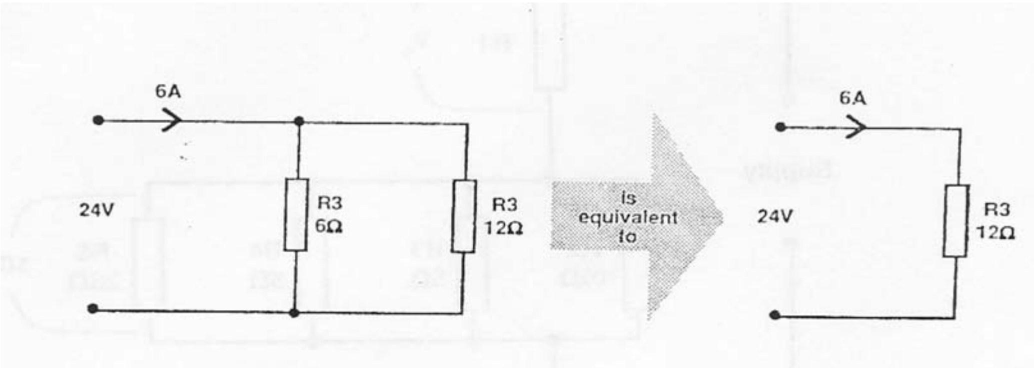
$$I_5 = \frac{V_2}{R_5} = \frac{100}{25} = 4A$$

$$I_T = I_2 + I_3 + I_4 + I_5$$
$$= 10 + 20 + 20 + 4$$
$$= 54A$$

Equivalent Resistance

There is still one more thing we can find out about a parallel circuit, its equivalent combined resistance

Equivalent resistance is the single resistance which could replace the ones in parallel and cause the same combined current to be drawn from the supply.

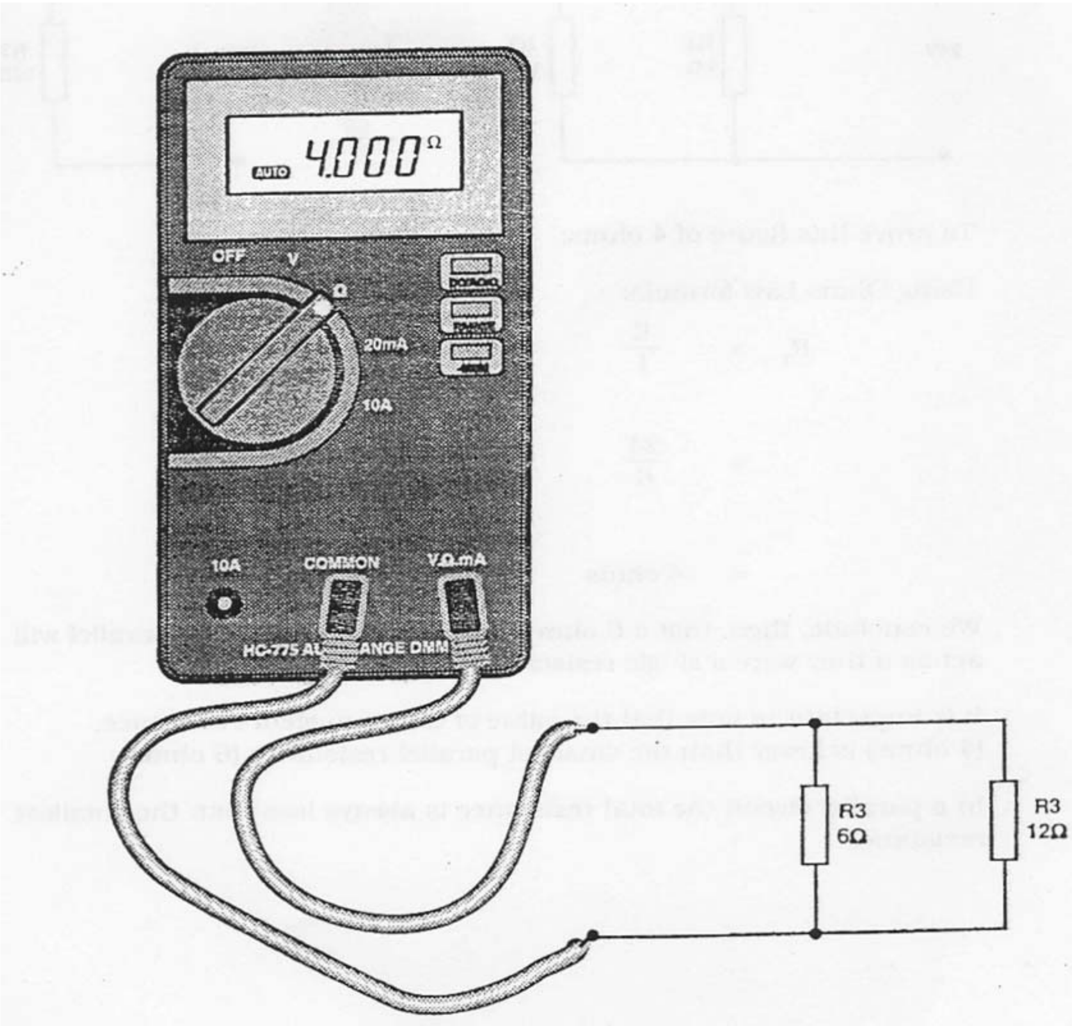


Experimentally it can be shown that two resistors of 6 ohms and
12 ohms in parallel could be replaced by a single resistor of four ohms
to take the current.

Calculation of Equivalent Resistance

If we connect an ohmmeter across the circuit, as shown below, it will measure the equivalent combined resistance of the 6 ohm and 12 ohm resistors in parallel.

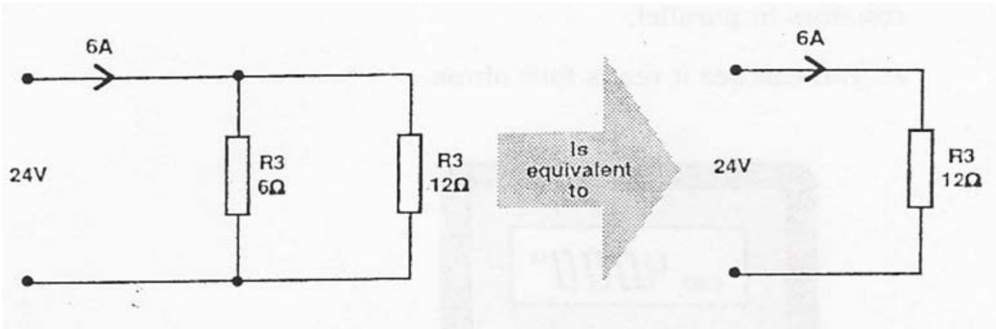
As you can see it reads four ohms



So, with parallel circuits you do **not** add the resistances.

To confirm the meter reading we can work out the equivalent resistance using Ohms Law.

We have already found the total current that would flow in the equivalent resistance and we know the voltage across it.



To prove this figure of 4 ohms:

Using Ohms Law Formula:

$$R_T = \frac{E}{I}$$

$$= \frac{24}{6}$$

$$= 4 \text{ ohms}$$

We conclude, then, that a 6 ohm and 12 ohm resistance in parallel will act as if they were a single resistance of 4 ohms.

It is important to note that the value of the equivalent resistance. (4 ohms) is lower than the smallest parallel resistance (6 ohms).

In a parallel circuit the total resistance is always less than the smallest resistance.

Method Two

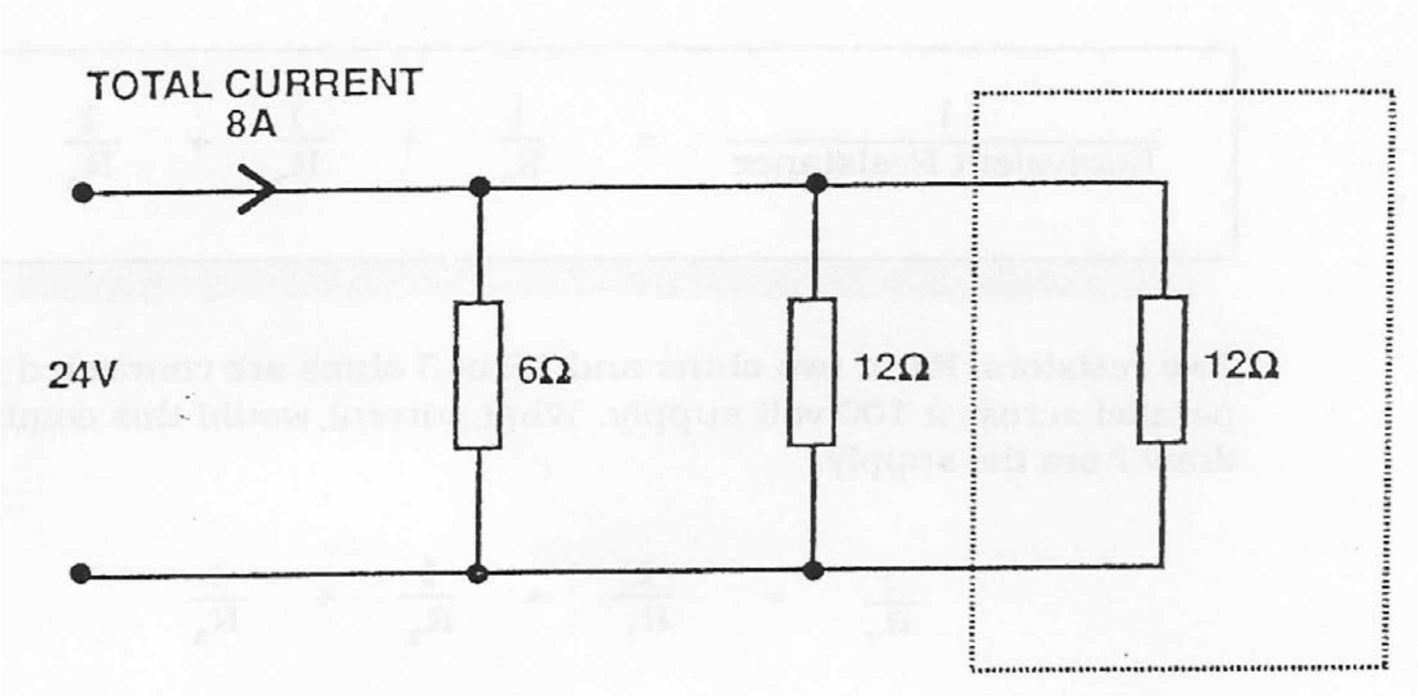
Another way of calculating the parallel resistance of a number of resistors in parallel is from the formula

$$\frac{1}{\text{Equivalent Resistance}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Two resistors, R_1 of two ohms and R_2 of 3 ohms are connected in parallel across a 100 volt supply- what current would this combination draw from the supply?

A Worked Example

- Study the circuit diagram below and compare it with the circuit on page 17



You can see that adding another 12 ohm resistor in parallel has caused the total current to increase to 8A.

Work out what the equivalent resistance is now.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{12} + \frac{1}{12}$$

$$\frac{1}{R_T} = \frac{4}{12}$$

$$R_T \quad 3\Omega$$

Or, to calculate using Ohms Law:

$$R_T = \frac{E}{I_T}$$

$$= \frac{24}{8}$$

$$= 3\Omega$$

The equivalent resistance was 4 ohms before the twelve ohms resistor was added in paral1el.

So, adding the resistor has *lowered* the equivalent resistance

For example, add another turnstile at a football ground and the crowd will move more quickly into the grounds, or to put it another way, the total stream of people experiences less resistance to movement by the addition of an extra turnstile

The table below gives the order in which you must work when dealing with simple parlal1el circuits where supply voltage and individual resistance values are given.

1 Determine the voltage drop across each parallel resistance.
The voltage drop across each resistance is the same.

2 Calculate the current in each parallel resistance,
Use Ohms Law formula:

$$I = \frac{V}{R}$$

The two known values being the voltage drop across the resistor and the resistor's resistance.

3 Determine the total current flowing in the circuit.
The total current is equal to the sum of the currents in the parallel resistors.

4 Calculate the Equivalent Resistance
Use Ohms Law formula:

$$I = \frac{V}{R}$$

The two known values being the voltage drop across the resistor, and $1_T$the total current.

To Add Fractions

In order to add or subtract fractions, (which often has to be done in order to solve parallel circuit problems) the following guidelines will be useful:

Write down the fractions to be added

For example:

$$\frac{1}{3} + \frac{1}{2}$$

Draw a line underneath them

$$\frac{1}{3} + \frac{1}{2}$$

here > ______

Below this line write the lowest common denominator (that is, the lowest number that all the denominators will divide into without a remainder). A common denominator may be found by multiplying all of the denominators together. It may not be the lowest common denominator but it can be used.

$$\frac{1}{3} + \frac{1}{2}$$

here > 6

A fraction has two parts = $\frac{\text{Numerator}}{\text{Denomiator}}$

Divide each denominator of the fractions to be added together, into the common denominator, and write the answer below it, that is, on the line drawn above the common denominator.

$$\frac{1}{3} + \frac{1}{2}$$

here > $\frac{2 \quad 3}{6}$

Add the numbers above the common denominator.

$$\frac{1}{3} + \frac{1}{2}$$

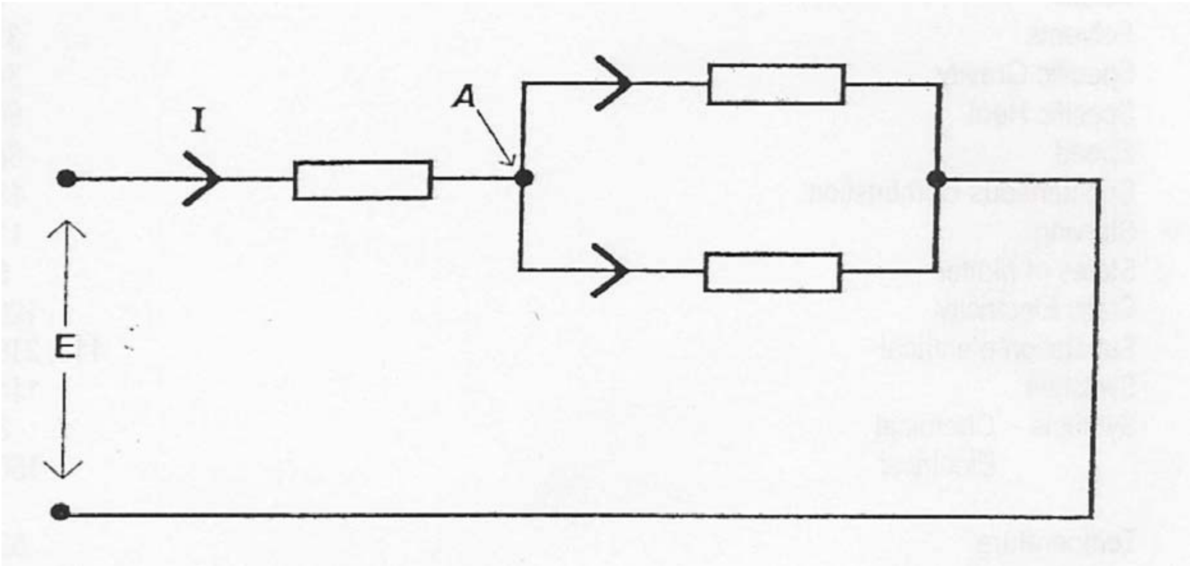
add these > $\frac{2 + 3}{6}$

So,

$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

Series Parallel Circuits

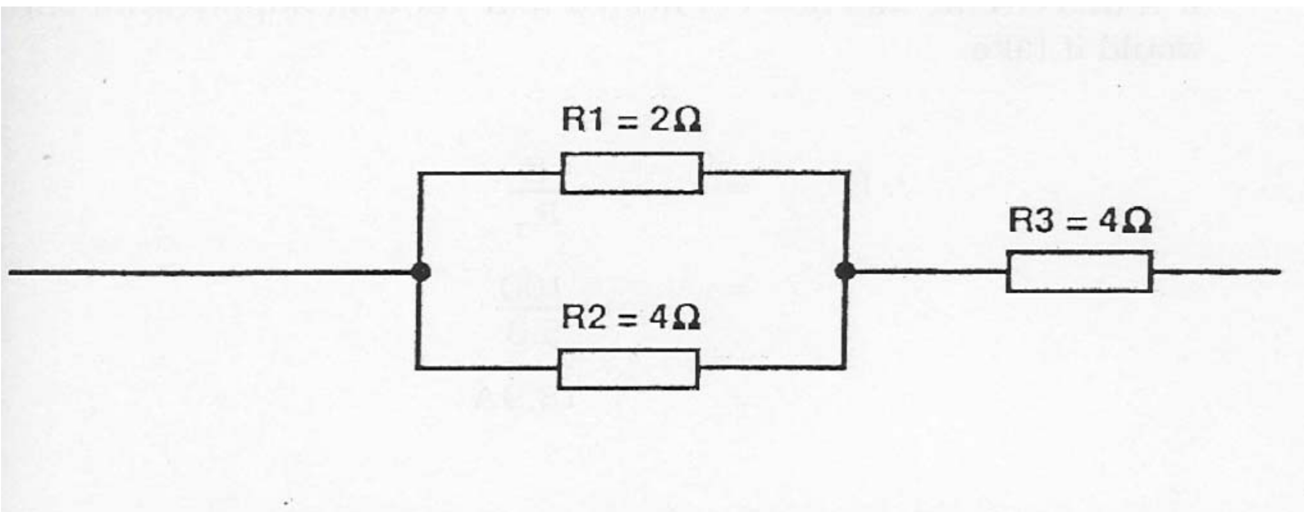
A circuit can be made up of resistances connected in series with one or more parallel combinations



In the above circuit current will flow through the series resistor and then divide at A and flow through both branches of the parallel combination. Because current has passed through the series resistor, a voltage drop will occur across it. Therefore the voltage across the parallel resistors will not be the same as the voltage applied to the circuit. It will be the e.m.f. voltage minus the voltage drop across the series resistor.

To calculate the total resistance in the circuit first reduce the parallel portion of the circuit to an equivalent resistance value, and then deal with the total circuit as a series circuit.

This equivalent resistance value is the value of resistance that will replace the paralleled resistors.



• Two resistors, R1 and R2 of two ohms and four ohms are connected in parallel and then connected in series with a resistor R3 of 4 ohms

1 Calculate the total circuit resistance.

Equivalent Resistance of the Parallel Branch $= R_P$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_P} = \frac{1}{2} + \frac{1}{4}$$

$$= \frac{3}{4}$$

$$R_P = \frac{4}{3}$$

$$= 1.3\Omega$$

Total Resistance of Circuit $= R_T$

$$R_T = R_3 + R_P$$

$$= 1.3 + 4$$

$$= 5.3\ \Omega$$

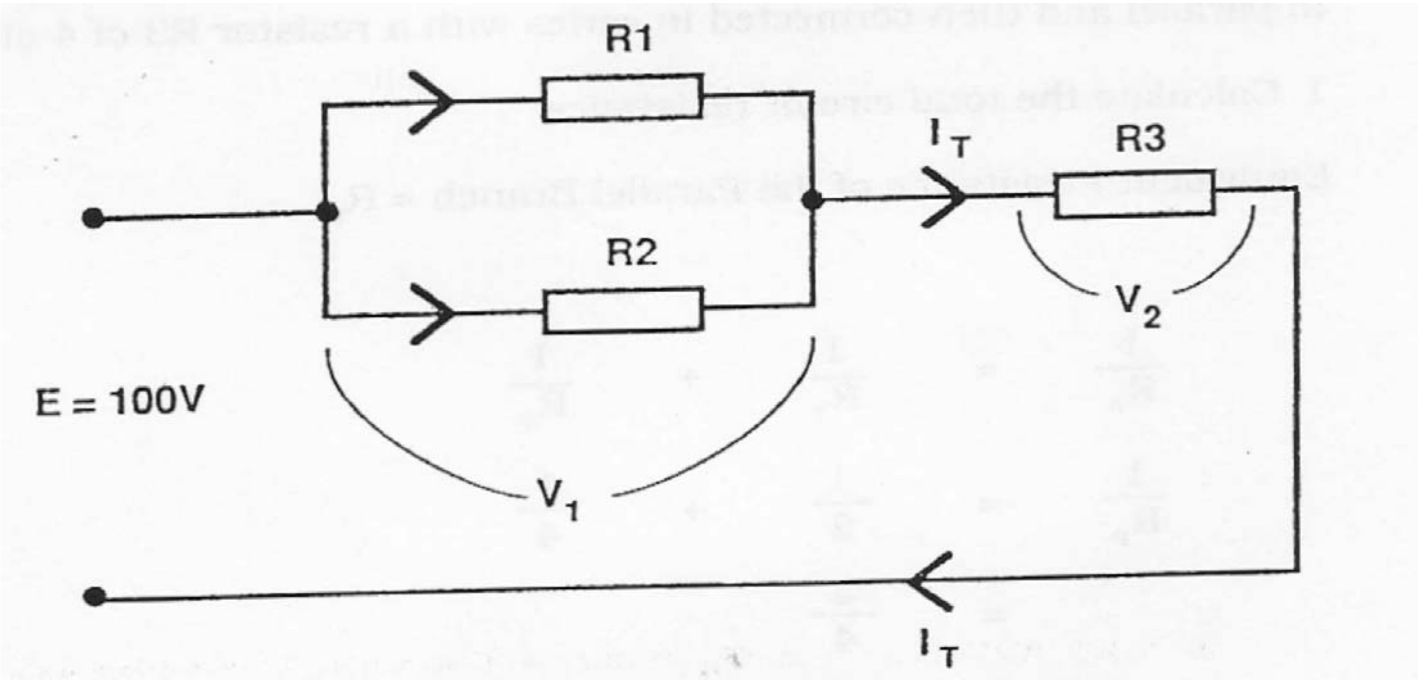
2 If this circuit was now connected to a 100 volt supply what current would it take.

$$IT = \frac{E}{RT}$$

$$= \frac{100}{5.3}$$

$$= 18.9A$$

2 What would then be the volt drop across each resistor.



V_1 = I_T x Equivalent R of R_1 and R_2

= 18.9 x 1.3

= 24.6V

V_2 = I_T x R_3

= 18.9 x 4

= 75.5V

V_2 = E - V1

= 100 - 24.6

= 75.4V

(The slight difference in the answers is due to accuracies in the decimal places but is not significant.)

Summary

This module has considered series circuits in which the resistors are connected end to end and the features of a series circuit are:

(a) The current is the same value throughout the circuit.

(b) 'The total resistance of a series circuit is the sum of the resistances.

(c) The voltage drop across each resistor can be calculated using Ohms law

i.e. Volt Drop = circuit current x resistance

(d) The volt drops across each resistor when added together equals the supply voltage (E).

'The parallel circuit was then considered in which two or more resistors are connected across the supply or between the same two points in a circuit.

A The volt drop across all resistors in parallel is the same.

B The total resistance of a parallel circuit R_T can be calculated from the formula:

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

C The current in each resistor can be calculated from the volt drop across the resistor divided by its resistance (note the volt drop across the resistors will be the same for all the resistors in the parallel branch). If the parallel branch is the only thing in the circuit the volt drop across it will be the same as the supply voltage E.

D The total circuit current enters the branch and then splits up to pass through the parallel resistors in proportion to their values. So the total circuit current is the sum of all the currents in the parallel bank.

The third type of circuit was the series parallel one in which some resistors are connected in series with parallel banks of resistors.

In the series/parallel circuit the things to be remembered are:

A The total resistance of the circuit is the sum of the resistors in series plus the equivalent resistances of all the parallel banks.

B The total current in the circuit can be calculated by dividing the supply voltage E by the total resistance of the circuit or by calculating all the currents in any parallel bank and adding them together, or dividing the volt drop across any series resistor by its resistance.

C The total voltage of the supply E can be calculated by adding all the volt drops in the circuit (remembering that each series resistor has a volt drop but only each parallel bank has one volt drop across it).